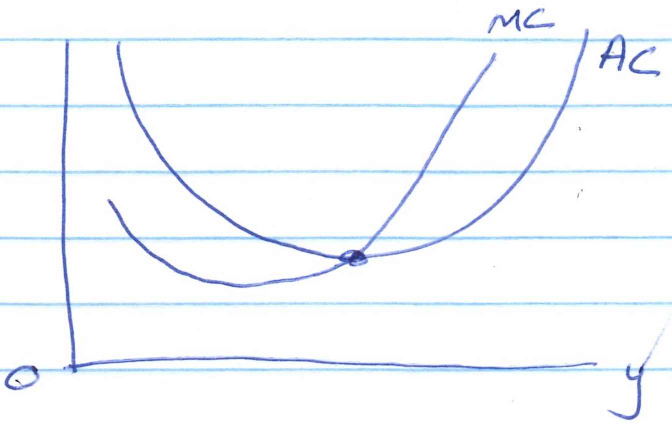
Econ 802
Midterm 2 Answers

Greg Dow November 2019

1. (a)



$$\frac{d\,[AC(y)]}{dy} = \frac{d\left[\frac{c(y)}{y}\right]}{dy} = \frac{c'(y)}{y} - \frac{c(y)}{y^2} = \frac{1}{y}\left[MC(y) - AC(y)\right]$$

So when AC is falling, $MC(y) < AC(y)$
when AC is at a min, $MC(y) = AC(y)$
when AC is rising, $MC(y) > AC(y)$

The U-shaped MC curve shown in the graph would be consistent with those results, although MC does not need to have this exact shape.

(b) Write the cost function as

$$c(w,y) = wx(y) - \lambda(y)\left[f[x(y)] - y\right]$$

where $x(y)$ is the optimal input vector for the output y. Note that this way of writing $c(w,y)$ is correct because $f[x(y)] = y$ for all y. Now differentiate with respect to y:

$$\frac{\partial c(w,y)}{\partial y} = w\frac{\partial x}{\partial y} - \frac{\partial \lambda}{\partial y}\left[\cancel{f}\,(x(y)) - y\right] - \lambda\frac{\partial f}{\partial x}\frac{\partial x}{\partial y} + \lambda$$

Reorganizing this gives

$$MC(y) = \frac{\partial c(w,y)}{\partial y} = \underbrace{\left[w - \lambda \frac{\partial f}{\partial x}\right]}_{=0 \text{ by FOC}} \frac{\partial x}{\partial y} - \frac{\partial \lambda}{\partial y} \underbrace{\left(f[x(y)] - y\right]}_{=0 \text{ by FOC}} + \lambda(y)$$

So $MC(y) = \lambda(y)$

This is related to the envelope theorem because we are differentiating the minimized level of expenditure with respect to a parameter and using the fact that indirect effects operating through the optimal choices $x(y)$ can be ignored.

(c) By definition, $e(x) = \dfrac{\sum_{i=1}^{n} \frac{\partial f(x)}{\partial x_i} \cdot x_i}{f(x)}$

At x^* we have $w_i = \lambda \dfrac{\partial f(x^*)}{\partial x_i}$ for all $i = 1..n$,

Substitution gives $e(x^*) = \dfrac{\sum_{i=1}^{n} \left(\frac{w_i}{\lambda}\right) x_i^*}{y^*} = \dfrac{1}{\lambda(y^*)} \dfrac{\sum_{i=1}^{n} w_i x_i^*}{y^*}$

Using $\lambda(y^*) = MC(y^*)$ and $\dfrac{\sum_{i=1}^{n} w_i x_i^*}{y^*} = \dfrac{c(w,y^*)}{y^*} = AC(y^*)$

we have $e(x^*) = \dfrac{AC(y^*)}{MC(y^*)}$

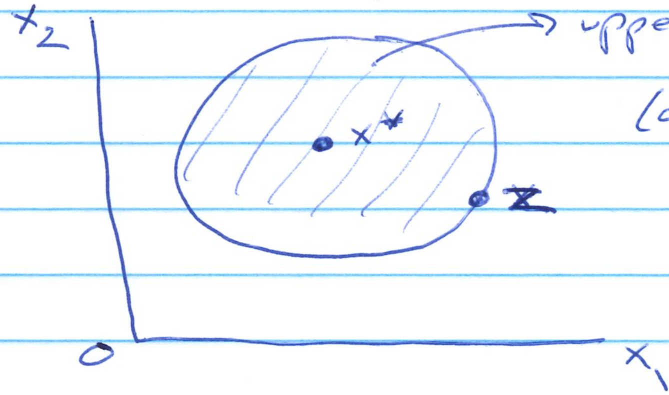
This is related to part (a) because when AC is falling we must have $AC(y) > MC(y)$ which implies $e(x) > 1$ so we have locally increasing returns to scale. Likewise when AC is rising we must have $AC(y) < MC(y)$ which implies $e(x) < 1$ so we have locally decreasing returns to scale.

2. (a) Weak monotonicity – No. This would require $x \succsim y$ whenever $x \geq y$. But if we start from the bliss point (set $y = x^*$), giving Connie more of anything makes her worse off.

Strong monotonicity – No. This would require $x \succ y$ whenever $x \geq y$ and $x \neq y$. But as before, giving her more of anything makes her worse off starting from $y = x^*$.

Local non-satiation – No. We can choose $x^* \in X$. Now choose any $\epsilon > 0$. There is no bundle y with $|x^* - y| < \epsilon$ such that $y \succ x$ (no bundle is better than the bliss point x^*).

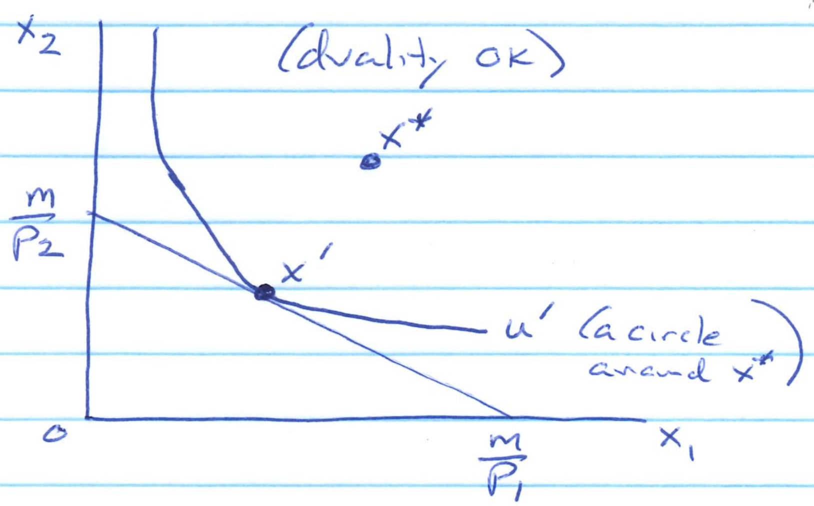
(b)

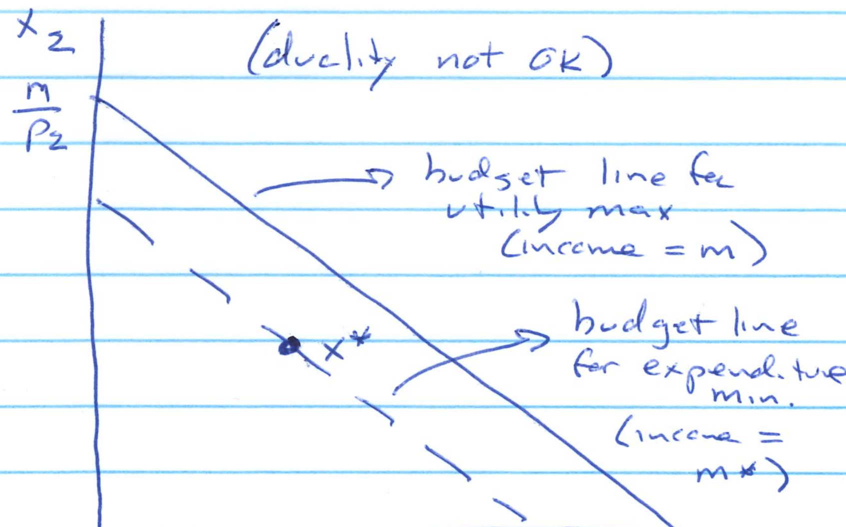


Convex? Yes. Choose any $x \succsim z$ and $y \succsim z$. Both x and y are in the upper contour set for z. Clearly any point on the line segment between x and y (with $x \succsim z$, $y \succsim z$) must also be in the upper contour set (at least as good as z).

Strictly convex? Yes. Now suppose $x \neq y$ and consider the points along the line segment between x and y that are strictly between, ie $tx + (1-t)y$ for $0 < t < 1$. All such points must be in the interior of the circle. Thus they are closer to x^* than z, and therefore strictly preferred to z.

2 (c) The answer is sometimes. Suppose the budget line passes below the bliss point x^*. Then the optimal bundle will be the point x' that is on the budget line and at the minimum distance from x^*. This gives a standard tangency solution and x' will also minimize the expenditure needed to achieve the utility level u'.



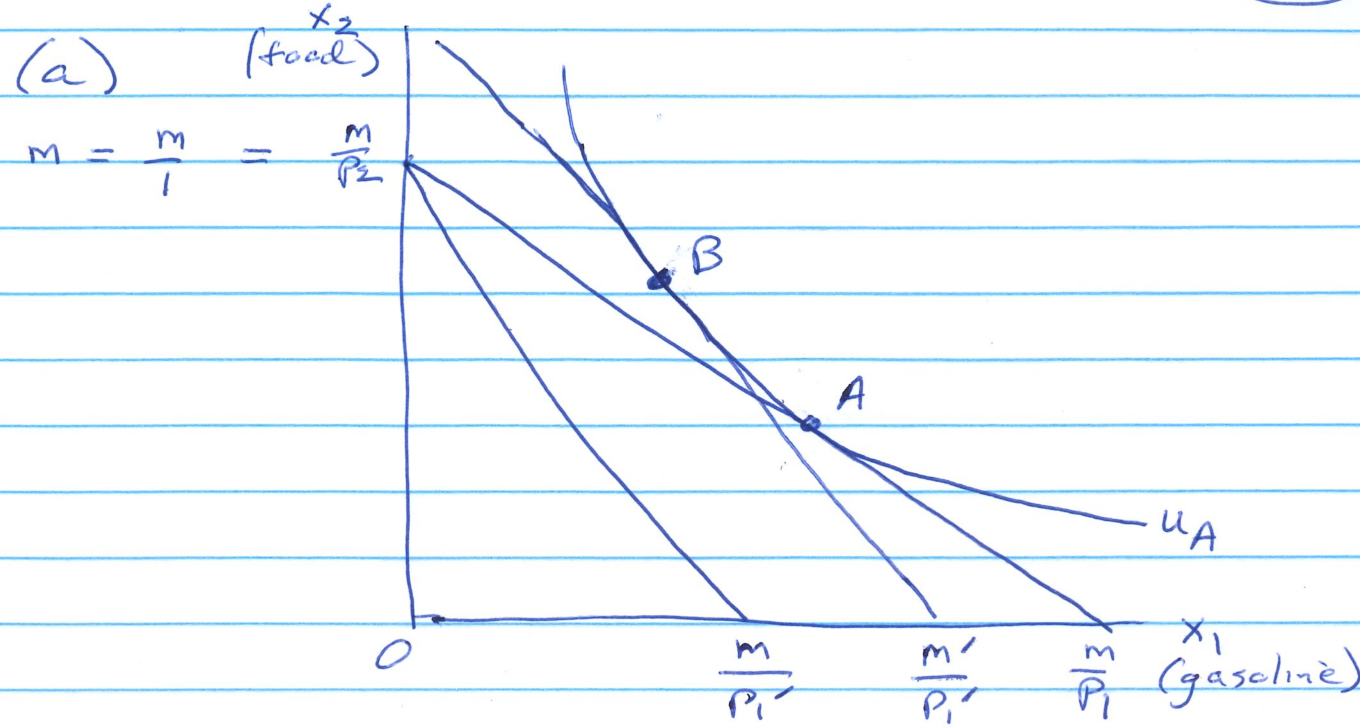


The problem arise when x^* is below the budget line as in the second graph. In that case, Connie chooses x^*. This would be her Marshallian demand vector. Let the resulting utility be $u^* = 0$ (no distance from the bliss point). If we try to minimize the expenditure needed to achieve $u^* = 0$ we get a budget line passing through x^* (see dashed line).

The problem is that the income level in the utility max problem (m) is not equal to the income level resulting from the expenditure min problem (m^*) so duality breaks down (even though x^* solves both problems)

Why? We don't have local non-satiation.

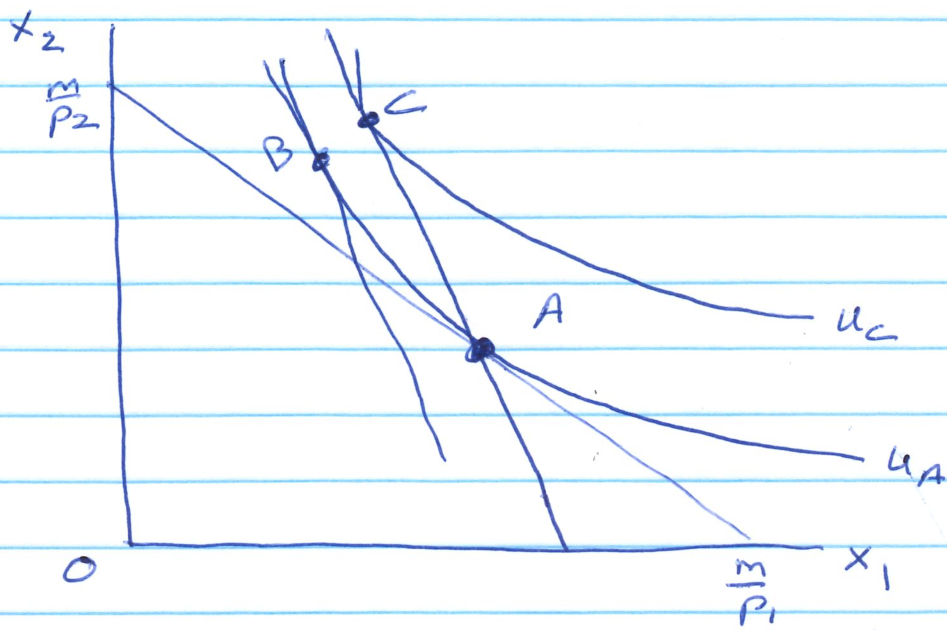
3 (a)



A is the original consumption bundle with income m and price p_1. If all we do is raise the price of gasoline to $p_1' > p_1$ we get a new steeper budget line with the same vertical intercept and the horizontal intercept $\frac{m}{p_1'}$. However, we are also going to raise income to a level $m' > m$ that puts Conrad back on the original indifference curve u_A so he is no worse off. The resulting budget line has the steeper slope due to p_1' but is tangent to u_A at a point like B. The net effect is that gasoline consumption definitely falls, because x_1 is smaller at B than at A. This must be true due strict quasi-concavity (it is just a negative substitution effect).

(b) Let the original price vector be $(p_1, 1)$ so the original expenditure level is $m = e(p_1, 1, u_A)$. We can compute m' as $m' = e(p_1', 1, u_A)$ (the minimum income needed to reach point B). The <u>additional</u> income needed is $m' - m = e(p_1', 1, u_A) - e(p_1, 1, u_A)$.

3(c)



The new price $p_1' > p_1$ gives a steeper budget line, and this new budget line must pass through point A in order for A to be affordable. Along the new budget line, Conrad chooses an optimal point like C with utility u_C. Although he does reduce gas consumption at C compared to A, the reduction is not as big as at point B from part (a). The reason is that gasoline is a normal good, so the parallel shift in the budget line implies more gas consumption at C than at B. Clearly Conrad prefers this policy because $u_C > u_A$ and the previous policy kept him on u_A.

4 (a) Use Shephard's Lemma:

$$\frac{\partial e(p,u)}{\partial p_1} = h_1(p,u) = \frac{u}{a} \quad \text{and} \quad \frac{\partial e(p,u)}{\partial p_2} = h_2(p,u) = \frac{u}{b}.$$

$$\frac{\partial h(p,u)}{\partial p} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This is symmetric (the off-diagonal elements are equal) and neg semidef (pre and post multiplying by the same vector always gives zero).

4 (b) Use Roy's identity:

$$x_1(p,m) = -\frac{\frac{\partial v(p,m)}{\partial p_1}}{\frac{\partial v(p,m)}{\partial m}} = \frac{-m(-1)\left[\frac{p_1}{a}+\frac{p_2}{b}\right]^{-2}\left(\frac{1}{a}\right)}{\left[\frac{p_1}{a}+\frac{p_2}{b}\right]^{-1}}$$

$$\Rightarrow \quad x_1(p,m) = \left(\frac{m}{a}\right)\left[\frac{p_1}{a}+\frac{p_2}{b}\right]^{-1}$$

$$x_2(p,m) = -\frac{\frac{\partial v(p,m)}{\partial p_2}}{\frac{\partial v(p,m)}{\partial m}} = \frac{-m(-1)\left[\frac{p_1}{a}+\frac{p_2}{b}\right]^{-2}\left(\frac{1}{b}\right)}{\left[\frac{p_1}{a}+\frac{p_2}{b}\right]^{-1}}$$

$$\Rightarrow \quad x_2(p,m) = \left(\frac{m}{b}\right)\left[\frac{p_1}{a}+\frac{p_2}{b}\right]^{-1}$$

(c) Try using the method $u(x) = \min_p v(p,1)$ s.t. $px = 1$

The Lagrangian is $L = \left[\frac{p_1}{a}+\frac{p_2}{b}\right]^{-1} \underbrace{+\lambda\left[p_1x_1+p_2x_2-1\right]}$

FOC: $(-1)\left[\frac{p_1}{a}+\frac{p_2}{b}\right]^{-2}\left(\frac{1}{a}\right)+\lambda x_1 = 0$

$(-1)\left[\frac{p_1}{a}+\frac{p_2}{b}\right]^{-2}\left(\frac{1}{b}\right)+\lambda x_2 = 0$

Note: I'm using the + sign here to ensure $\lambda > 0$

2 problems arise: ① if we divide one equation by the other, we get $\frac{\left(\frac{1}{a}\right)}{\left(\frac{1}{b}\right)} = \frac{x_1}{x_2}$, but a and b are constants, so what happens if x has $\frac{x_1}{x_2} \neq \frac{b}{a}$?

② we want to solve for optimal prices here, but can only solve for $\left[\frac{p_1}{a}+\frac{p_2}{b}\right]$; no unique solution for (p_1, p_2)

The true utility function is Leontief: $u(x) = \min\{ax_1, bx_2\}$
You can guess this from the fact that the substitution effects in part (a) were all zero.
The underlying problem is that $u(x)$ is not differentiable, so the usual procedure for calculating inverse demand functions does not work here.

5 (a) The Lagrangian is $\ln c + \ln L - \lambda(pc + wL - m)$

FOC: $\frac{1}{c} - \lambda p = 0 \Rightarrow 1 = \lambda pc$
$\frac{1}{L} - \lambda w = 0 \Rightarrow 1 = \lambda wL$
$\lambda = \frac{1}{pc} \Rightarrow 1 = \frac{wL}{pc} \Rightarrow wL = pc$

Using the budget constraint, $pc + wL = 2pc = m$
$pc + wL = 2wL = m$

So the Marshallian demands are $c(p, w, m) = \frac{m}{2p}$

$L(p, w, m) = \frac{m}{2w}$

$V(p, w, m) = \ln\left(\frac{m}{2p}\right) + \ln\left(\frac{m}{2w}\right)$

(b) labor supply is $H(p, w, m) = T - L(p, w, m) = T - \frac{m}{2w}$

However, with $r = 0$ we have $m = wT + r = wT$
Thus $L(p, w, m) = \frac{m}{2w} = \frac{wT}{2w} = \frac{T}{2}$ which is constant.

And so $H = T - \frac{T}{2} = \frac{T}{2}$ is also constant.

The prices (p, w) have no effect on Constantine's labor/leisure decisions. The reason is that his time endowment is in the budget constraint, and substitution effects are exactly cancelling out against effects of prices on his income m. For example for $w\uparrow$ we get a higher "price" of leisure which tends to reduce L. But he is a net supplier of time to the labor market, and a higher wage also increases his income. Because leisure is a normal good, this tends to increase L. (The story is similar for changes in p). Because L remains constant, H must also remain constant (they add up to T, which is constant). Therefore Constantine has a zero elasticity of labor supply with respect to the wage.

5 (c) Notice that the indirect utility function

$$v_i(p, m_i) = \frac{m_i}{2(pw)^{1/2}} \quad \text{for person } i$$

is in the Gorman form; we have

$$v_i(p, m_i) = a_i(p) + b(p)\, m_i \quad \text{where } a_i(p) = 0 \text{ for all } i$$

and $b(p) = \dfrac{1}{2(pw)^{1/2}}$

for all i.

Furthermore $b(p)$ does not depend on i. Therefore, aggregation is possible and aggregate demands for consumption and leisure only depend on aggregate income.

This is easy to check using your results in part (a). The individual Marshallian demands are

$$c_i(p, w, m_i) = \frac{m_i}{2p} \quad \text{and} \quad L_i(p, w, m_i) = \frac{m_i}{2w}$$

The aggregate demands are

$$C(p, w, m_1 \ldots m_n) = \sum_i c_i(p, w, m_i) = \frac{\sum_i m_i}{2p} = \frac{M}{2p} = C(p, M)$$

$$L(p, w, m_1 \ldots m_n) = \sum_i L_i(p, w, m_i) = \frac{\sum_i m_i}{2w} = \frac{M}{2w} = L(w, M)$$

Because aggregate demand for leisure does not depend on income distribution, the same is true for aggregate labor supply.